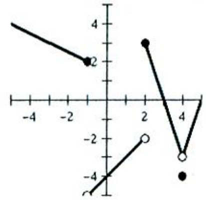
The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 3 PTS**



[a] $\lim\limits_{x \to 4} \dfrac{x^2}{2 + f(x)}$

$$= \frac{\lim\limits_{x \to 4} x^2}{\lim\limits_{x \to 4} 2 + \lim\limits_{x \to 4} f(x)} \quad 1$$

$$= \frac{16}{2 + -3} = \frac{16}{-1} = -16 \quad 1$$

[b] $\lim\limits_{x \to -1^-} f(x)$

$$= 2 \quad 1$$

Prove that $\lim\limits_{x \to 0} x^4 \cos \dfrac{1}{x^2} = 0$. **SCORE: _____ / 3 PTS**

$$-x^4 \le x^4 \cos \frac{1}{x^2} \le x^4, \quad 1$$

$$\lim_{x \to 0} -x^4 = 0 \quad \tfrac{1}{2}$$

$$\lim_{x \to 0} x^4 = 0 \quad \tfrac{1}{2}$$

OK IF TOGETHER IN A COMPOUND EQUALITY

SO $\lim\limits_{x \to 0} x^4 \cos \dfrac{1}{x^2} = 0$ BY SQUEEZE THEOREM 1

If $\lim\limits_{r \to -1} \dfrac{7 + ar - r^5}{1 + r}$ exists, find the value of a. **SCORE: _____ / 2 PTS**

SINCE $\lim\limits_{r \to -1} (1 + r) = 0$, THE ORIGINAL LIMIT EXISTS ONLY

IF $\lim\limits_{r \to -1} (7 + ar - r^5) = 0$ IE. $7 - a + 1 = 0$, 1

OK IF SIMPLIFIED

$a = 8$, 1

Using complete sentences and proper mathematical notation, write the formal definition of "vertical asymptote". **SCORE: _____ / 2 PTS**

f HAS A VERTICAL ASYMPTOTE AT a IFF

$\lim_{x\to a^+} f(x)=\infty$ OR $\lim_{x\to a^+} f(x)=-\infty$ OR $\lim_{x\to a^-} f(x)=\infty$ OR $\lim_{x\to a^-} f(x)=-\infty$

GRADED BY ME

Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 7 PTS**

[a] $\lim_{y\to -2} \frac{y^2-3y-10}{2y^2-3y-14}$ $\frac{0}{0}$

$= \lim_{y\to -2} \frac{(y+2)(y-5)}{(y+2)(2y-7)}$

$= \boxed{\lim_{y\to -2} \frac{y-5}{2y-7}}$ 1

$= \frac{-7}{-11} = \boxed{\frac{7}{11}}$ $\frac{1}{2}$

[b] $\lim_{b\to 5} \frac{b-\sqrt{b+20}}{10-2b}$ $\frac{0}{0}$

$= \boxed{\lim_{b\to 5} \frac{b^2-b-20}{(10-2b)(b+\sqrt{b+20})}}$ 1

$= \lim_{b\to 5} \frac{(b-5)(b+4)}{-2(b-5)(b+\sqrt{b+20})}$

$= \boxed{\lim_{b\to 5} \frac{b+4}{-2(b+\sqrt{b+20})}}$ 1

$= \frac{9}{(-2)10}$

$= \boxed{-\frac{9}{20}}$ $\frac{1}{2}$

[c] $\lim_{t\to -3} \frac{\frac{8}{1-t}-\frac{1}{t+2}}{t^2+9}$

$= \frac{\frac{8}{4}-\frac{1}{-1}}{9+9}$

$= \frac{2+1}{18}$

$= \frac{3}{18} = \boxed{\frac{1}{6}}$ 1

[d] $\lim_{x\to 4} f(x)$ where $f(x) = \begin{cases} 2x+1, & \text{if } x<0 \\ 1-x, & \text{if } 0<x<4 \\ x-7, & \text{if } x>4 \end{cases}$

$\lim_{x\to 4^-} f(x) = \boxed{\lim_{x\to 4^-} (1-x) = -3}$ REQUIRED $\frac{1}{2}$

$\lim_{x\to 4^+} f(x) = \boxed{\lim_{x\to 4^+} (x-7) = -3}$ REQUIRED $\frac{1}{2}$

SO $\boxed{\lim_{x\to 4} f(x) = -3}$ 1